1.5.4 Quantificational ambiguities

One benefit of learning a logical language is its use for identifying and resolving ambiguities in natural language. In contrast to natural language, the language of predicate logic does not contain any ambiguity. Names refer to exactly one item in the domain, each n-place predicate picks out a set of n-tuples in the domain, operators have a fixed meaning, and the scope of the various operators is precise. In contrast, natural language is rife with ambiguity. In some cases, the existence of ambiguity influences our evaluation of an argument. For example, consider the following argument:

P1. All stars are in outer space.
P2. Socrates is a star.
C. Therefore, Socrates is in outer space.

In the above example, the word "star" is ambiguous. It has two distinct meanings. A "star" can refer to a celestial body that produces light and heat, or it can refer to a celebrity. Noting this ambiguity, consider three different ways in which the argument can be evaluated.

First, we take "star" to have the *same* meaning in both premises. For the sake of the example, let's suppose it means "a celestial body that produces light and heat". If this is the case, then the argument is valid. However, notice that the validity of the argument comes at a cost. If "star" means "a celestial body that produces light and heat", then P2 is false. Socrates is not a star in this sense (he is a philosophical celebrity). So, the argument is valid but unsound.

Second, we take "star" to have *different* senses in the two premises. In P1, "star" refers to a class of celestial bodies that produce light and heat. In P2, "star" refers to a celebrity. If this is the case, then the premises of the argument are true, but the argument is invalid. This way of evaluating the argument is certainly the most natural and reasonable.

Third, when evaluating the argument for validity, we take "star" to have the same meaning in both premises and so the argument is valid. However, when we evaluate the truth of the premises, we take the different instances of "star" to have different meanings (in P1 "celestial body", in P2 "celebrity"). If this is the case, then the argument has true premises. The argument then is said to be sound (valid and true premises), although this comes at the cost of an inconsistency since we are saying the two occurrences of "star" have different meanings but also the same meaning.

1. Approach 1: Argument is valid but at least one premise is false.

2. Approach 2: Argument is invalid but has true premises.
3. Approach 3: Argument is sound (valid and true premises), but one has to accept an inconsistency.

It is clear then that the existence of ambiguity in natural language can have an impact on the evaluation of arguments. Insofar as one of the above approaches is more plausible than the others, it is important to be able to identify and resolve ambiguities in natural language.

What sort of ambiguities exist in natural language that can be revealed using predicate logic? Let's consider a type of ambiguity that we will call a "quantificational ambiguity".

Definition 1.11

A *quantificational ambiguity* is an ambiguity that arises when a sentence is ambiguous about the presence or scope of a quantifier.

There are at least two types of quantificational ambiguities: quantificational presence ambiguities and quantificational scope ambiguities. The first type involves a sentence where the sentence suggests the presence of a quantifier, but it is ambiguous as to which quantifier is present. The second type involves a sentence where the sentence is ambiguous as to the scope of a quantifier.

Quantificational presence ambiguities

Let's begin with quantificational presence ambiguities. One simple example involves the following sentence:

> Birds fly.

How should we translate this sentence into predicate logic? There appear to be two different possibilities. The first possibility is that "Birds fly" means "*All* birds fly" and so we would translate the sentence using the universal quantifier as follows: $(\forall x)(Bx \rightarrow Fx)$. The second possibility is that "Birds fly" means "*Some* birds fly" and so we would translate the sentence using the existential quantifer as follows: $(\exists x)(Bx \wedge Fx)$.

$$\begin{array}{ccc} & \text{Birds fly} & \\ \swarrow & & \searrow \\ \text{All birds fly} & & \text{Some birds fly} \\ (\forall x)(Bx \rightarrow Fx) & & (\exists x)(Bx \wedge Fx) \end{array}$$

Our attempt to translate "Birds fly" into predicate logic reveals that the sentence is not explicit about which quantifier is present. Since both interpretations of the sentence are possible and these intepretations are not equivalent, an ambiguity is present. As mentioned, this type of ambiguity may impact how arguments are evaluated. Consider the following argument:

P1: Birds fly.
C: Therefore, penguins fly.

If "birds fly" expresses "all birds fly", then the above argument is valid but unsound. P1 would be false since penguins are birds, but do not fly. In contrast, if "birds fly" expresses "some birds fly", then while P1 is true, the argument is invalid. Finally, someone might contend that the argument is sound (valid and has true premises), but they would do so at the cost of inconsistency: they would be saying that "birds fly" expresses "all birds fly" and "some birds fly".

Our previous example involving birds is straightforward and few people fail to recognize that there is something fishy about the sentence itself. In what follows, let's consider a similar, but real-life example of a quantificationally ambiguous sentence. Consider the following sentence:

> The truth of the matter is that pretty much anywhere in the world men tend to think that they are much smarter than women.[qtd in 1]

Similar to the previous example, the sentence "men think that they are smarter than women" is quantificationally ambiguous. First, it is indeterminate about how many man think they are smarter than women (all, some, most, etc.). To resolve the ambiguity, we might try to resolve it by prefixing a different quantificational expression ("all" or "some") to each sentence.

1. *All* men think that they are smarter than women.
2. *Some* men think that they are smarter than women.

In prefixing a quantifier to "men think they are smarter than women", we have made explicit the ambiguity concerning the *presence* of a quantifier. The sentence itself does not contain any quantificational expression, but the interpretation of the sentence requires the addition of some quantificational expression. However, note that even after we have made explicit the ambiguity concerning the presence of a quantifier, there still remains *another* quantificational ambiguity. That is, not only is there an ambiguity concerning *how many men* think a certain way, but there is also ambiguity concerning *what* they think. To keep things somewhat simple, let's suppose that the author is attempting to characterize what *all* rather than *some* or *most* men think. We can then ask, *what* is it that each man thinks? Each man thinks "men are smarter than women". This sentence however has two quantificational presence ambiguities since "men are smarter than women" can mean any of the following:

1. *All* men are smarter than *all* women.
2. *All* men are smarter than *some* women.
3. *Some* men are smarter than *all* women.
4. *Some* men are smarter than *some* women.

Each of these sentences has a corresponding translation in **RL** and each of these translations is distinct.

1. $(\forall x)(Mx \rightarrow (\forall y)(Wy \rightarrow Sxy))$
2. $(\forall x)(Mx \rightarrow (\exists y)(Wy \wedge Sxy))$
3. $(\exists x)(Mx \wedge (\forall y)(Wy \rightarrow Sxy))$
4. $(\exists x)(Mx \wedge (\exists y)(Wy \wedge Sxy))$

Making the quantificational ambiguity that is present in this sentence explicit is helpful for evaluating arguments containing this sentence. Consider the following argument:

- P1: Men think that they are smarter than women.
- C: Men are chauvinistic.

Depending on how P1 and P2 are interpreted in the above argument influences how the argument is evaluated. For example, let's consider just two different interpretations of P1 and P2, noting how it impacts the truth of the premises and the argument's validity:

1. If P1 is read as "all men think that all men are smarter than all women" and C is read as "all men are chauvinistic", then the argument is valid but unsound due to the falsity of P1.
2. If P1 is "some men think they are smarter than some women" and C is read as "all men are chauvinistic", then P1 is true but the argument is invalid.

Quantificational scope ambiguity

The first type of quantificational ambiguity we considered involved sentences that were ambiguous about the *presence* of a quantifier. The interpretation of the sentence requires the addition of some quantificational expression (e.g., "some", "all", "many") but the sentence itself does not contain any such expression. The absence of the quantifier gives rise to the ambiguity. Let's now turn to a second type of quantificational ambiguity. This second type concerns sentences that are ambiguous about the scope of a quantifier. Consider the following sentence:

Everyone loves someone.

This sentence is ambiguous between two readings. The first reading is that everyone loves at least one (not necessarily the same) person. In **RL**, we can translate this reading as $(\forall x)(\exists y)Lxy$. To illustrate, imagine three people: Jon, Tek, and Liz. This reading of the sentence is true in the following scenario: Jon loves Tek, Tek loves Liz, and Liz loves Jon. Notice that each person loves some person, although no person is loved by everyone. The second reading is that there is at least one person who is loved by everyone. In **RL**, we can translate this reading as $(\exists x)(\forall y)Lxy$. To illustrate, imagine three people: Jon, Tek, and Liz. This reading of the sentence is true in this scenario provided there exists a person (e.g., Tek) who is loved by Jon, Tek, and Liz.

$$(\forall x)(\exists y)Lxy \longleftarrow \text{Everyone loves someone.} \longrightarrow (\exists x)(\forall y)Lxy$$

What we see then is that the sentence "everyone loves someone" is ambiguous about the scope of the quantifiers. In the first reading, the universal quantifier has wide scope. In the second reading, the existential quantifier has wide scope.

Let's consider another example of a quantificational scope ambiguity. Our previous example involved a case where it was ambiguous whether the universal or existential quantifier had wide scope. In this example, we will consider a case where it is ambiguous whether the negation or the universal quantifier has wide scope. To set up this type of ambiguity, let's consider the following sentence:

Every student passed.

The translation of this sentence into **RL** is straightforward. Let Sx express the English expression "x is a student" and let Px express the English expression "x passed the exam". The sentence is then translated as $(\forall x)(Sx \to Px)$. With this translation in mind, let's consider the following sentence:

Every student did ***not*** pass.

This sentence is ambiguous between two readings. First, the sentence can be read as "it is not the case that every student passed". On this reading, the sentence is translated as $\neg(\forall x)(Sx \to Px)$, the negation operator having wide scope. Equivalently, this sentence says that "there is a student who did not pass", translated as $(\exists x)(S \wedge \neg Px)$. Second, the sentence can be read as "no student passed". On this reading, the sentence is translated as $(\forall x)(Sx \to \neg Px)$. The negation operator having narrow scope. Equivalently, this sentence says that "there does not exist a student that passed", translated as $\neg(\exists x)(Sx \wedge \neg Px)$.

$$\neg(\forall x)(Sx \to Px) \longleftarrow \text{Every student did not pass.} \longrightarrow (\forall x)(Sx \to \neg Px)$$

It is sometimes remarked that both of these readings are not suggested by the sentence. That is, the sentence "every student did not pass" only refers to one of these readings. To see how the sentence is capable of suggesting both readings, consider the following two scenarios. First, suppose that a group of students are standing around comparing their scores on their logic exam. Tek begins, saying that he passed the exam with flying colors. He says he received a 100%. Liz also says that she passed the exam. She says that it actually was one of the easiest exam she has ever taken. With a sad look on her face, the professor informs the students that while she's happy they did so well, it would be a good idea not to be too vocal about their scores since "every student did not pass the exam." On this reading, the sentence "every student did not pass the exam" means "***it is not the case that*** every student passed the exam": $\neg(\forall x)(Sx \to Px)$.

Second, suppose again that a group of students are standing around comparing their scores on their logic exam. Tek begins, complaining that this exam was really hard, noting he failed. Liz also says that she failed. She says that the exam was actually one of the hardest exams she has ever taken. With a sad look on her face, the professor apologizes to the students. The professor says that the exam was indeed very difficult and that "Every student did not pass the exam." On this reading, the sentence means "***no*** student passsed the exam.".

In this section, we considered how ambiguity can arise in natural language sentences and impact how arguments are evaluated. In particular, we considered two types of quantificational ambiguities: quantificational presence ambiguities and quantificational scope ambiguities. The first type involves a sentence where the sentence suggests the presence of a quantifier, but it is ambiguous as to which quantifier is present. The second type involves a sentence where the sentence is ambiguous as to the scope of a quantifier.

Exercise 13.

Identify the types of quantificational ambiguity present in the following sentences (involving presence or involving scope). What are the two readings of the sentence? Translate each reading into **RL**.

1. Criminals are bad.
2. All criminals are not greedy.
3. Smokers are not bad.
4. All fetuses are not persons.
5. Athletes are not heroes.