

## HANDOUT #3 – PROPOSITIONAL LOGIC – TRUTH TABLES

### Valuation ( $v$ )

A valuation (or truth-value assignment) is an assignment of a truth value (either T or F) to a proposition.

$$v(P)=T, v(R)=F$$

The truth value of complex propositions is determined by the truth value of the propositional letters that compose the propositions and truth-functional rules associated with truth-functional operators.

Examples:

If  $v(P)=T$ , then  $v(\neg P)=F$

If  $v(P)=T$  and  $v(R)=F$ , then  $v(P \wedge R)=F$

If  $v(P)=T$ , and  $v(R)=F$ , then the  $v(P \vee R)=?$

### Truth Tables: A More Graphical Method

Suppose,  $v(P)=T$ , and  $v(R)=F$

*Step #1:* Write out the formula or set of formulas you want to test.

P	$\rightarrow$	$\neg$	R

*Step #2:* Put truth values below propositional letters

P	$\rightarrow$	$\neg$	R
T			F

*Step #3:* Start from the truth-functional operators with the *least* scope, determine the truth value of more complex subformula, working your way to determining the truth value of the main operator.

P	$\rightarrow$	$\neg$	R
T		<b>T</b>	F

Finally,

P	$\rightarrow$	$\neg$	R
T	T	<b>T</b>	F

The truth value of the complex proposition  $P \rightarrow \neg R$  is the truth value under the main operator in the above table.

## Truth Tables for Propositional Forms

In the above examples, we are given the truth values of the propositional letters that compose the complex propositions. However, we can also represent the conditions under which a proposition is true (false) given *every* valuation of the propositional letters. For example, consider the following proposition:  $\neg P \vee \neg R$ .

*Step #1:* Write out all of the propositional letters in the formula in a separate column and the formula or formulas you want to test to the right of it:

P	R	$\neg P \vee \neg R$
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*Step #2:* consider the different possible truth value assignments:<sup>1</sup>

P	R	$\neg P \vee \neg R$
T	T	
T	F	
F	T	
F	F	

*Step #3:* starting at row 1, for each row, write the truth values under the corresponding letter in the row.

P	R	$\neg$	P	$\vee$	$\neg$	R
T	T		T			T
T	F		T			F
F	T		F			T
F	F		F			F

*Step #4:* Start from the truth-functional operators with the **least scope** to the operators with the **most scope** (main operator), determine the truth value of formula upon which these operators operate upon, and work your way to determining the truth value of the **main operator**.

P	R	$\neg$	P	$\vee$	$\neg$	R
T	T	<b>F</b>	T		<b>F</b>	T
T	F	<b>F</b>	T		<b>T</b>	F
F	T	<b>T</b>	F		<b>F</b>	T
F	F	<b>T</b>	F		<b>T</b>	F

P	R	$\neg$	P	$\vee$	$\neg$	R
T	T	F	T	F	F	T
T	F	F	T	T	T	F
F	T	T	F	T	F	T
F	F	T	F	T	T	F

<sup>1</sup> The number of rows you need to construct is determined by the number of propositional letters:  $2^n$  where  $n$ =number of propositional letters.

The table above shows under what valuation of the propositional letters that  $\neg P \vee \neg R$  is true (or false).

**Classroom Practice**

1.  $P \rightarrow \neg R$
2.  $(P \wedge R) \rightarrow R$
3.  $\neg \neg (P \leftrightarrow R) \vee Z$

**Truth-Table Analysis**

A **decision procedure** is a mechanical method that determines in a finite number of steps whether a proposition, set of propositions, or argument has a certain logical property (one of these being *whether or not an argument is deductively valid!*). Truth tables are a type of decision procedure as they can be used to test whether propositions, sets of propositions, or arguments have certain logical properties.

*For Propositions (Contingency, Tautology, Contradiction)*

<b>Tautology</b>	A proposition <b>P</b> is a <i>tautology</i> if and only if <b>P</b> is true under every valuation. A truth table for a tautology will have all T's under its main operator (or in the case of no operators, under the propositional letter).
<b>Contradiction</b>	A proposition <b>P</b> is a <i>contradiction</i> if and only if <b>P</b> is false under every valuation. A truth table for a contradiction will have all F's under its main operator (or in the case of no operators, under the propositional letter).
<b>Contingency</b>	A proposition <b>P</b> is a <i>contingency</i> if and only if <b>P</b> is neither always false under every valuation nor always true under every valuation. A truth table for a contingency will have at least one T and at least one F under its main operator (or in the case of no operators, under the propositional letter).

*Classroom Examples*

1.  $\neg P \rightarrow \neg P$
2.  $(P \wedge \neg P) \wedge Q$
3.  $P \leftrightarrow \neg R$

*For Sets of Propositions (Consistency and Equivalence)*

<b>Equivalence</b>	A pair of propositions <b>P</b> , <b>Q</b> is <i>logically equivalent</i> if and only if <b>P</b> and <b>Q</b> have identical truth values under every valuation. In a truth table for an equivalence, there is no row on the truth table where one of the pair <b>P</b> has a different truth value than the other <b>Q</b> .
<b>Consistency</b>	A set of propositions <b>{P, Q, R, ... Z}</b> is <i>logically consistent</i> if and only if there is at least one valuation where <b>P, Q, R, ... Z</b> are true. A truth table shows that a set of propositions is consistent when there is at least one row on the truth table where <b>P, Q, R, ... Z</b> are all true.

*Classroom Examples*

1.  $\neg P \wedge \neg R, \neg(P \vee R)$ , equivalent?
2.  $P, P \rightarrow R, R \vee P$ , consistent?
3.  $P \vee R, \neg R, \neg P$ , consistent?

*For Arguments (Validity)*

The single turnstile: ‘ $\vdash$ ’. Indicates the presence of an argument: the propositions to the *left* of the turnstile are the **premises** while the proposition to the *right* of the turnstile is the **conclusion**. For example, the turnstile in the following

$$P \wedge R \vdash R$$

indicates the presence of an argument where ‘ $P \wedge R$ ’ is the premise and ‘ $R$ ’ is the conclusion.

<b>Validity</b>	An argument $P, Q, \dots, Y \vdash Z$ is <i>valid</i> in PL if and only if it is impossible for the premises to be true and the conclusion false. A truth table shows that an argument is valid if and only if there is no row of the truth table where the premises are true and the conclusion is false.
<b>Invalidity</b>	An argument $P, Q, \dots, Y \vdash Z$ is <i>invalid</i> in PL if and only if it is possible for the premises to be true and the conclusion false. A truth table shows that an argument is invalid if and only if there is a row of the truth table where the premises are true and the conclusion is false.

*Classroom Examples*

1.  $P, P \rightarrow R \vdash R$
2.  $\neg P, \neg P \vee R \vdash R$

***Classroom Examples***

*Propositions*

1.  $P \rightarrow (P \vee Q)$
2.  $\neg \neg P \wedge P$
3.  $\neg(P \vee \neg R)$

*Sets of Propositions*

1.  $\neg P \rightarrow R, R \rightarrow \neg P$ , test for equivalence
2.  $\neg \neg P \rightarrow R, R \rightarrow Z, Z$ , test for consistency
3.  $P \vee R, P \rightarrow R, P \leftrightarrow R, P \wedge R$ , test for consistency

*Arguments*

1.  $P \vee R, \neg R, P \rightarrow R \vdash R$
2.  $J \rightarrow C, \neg C \vdash \neg J$
3.  $J \leftrightarrow C, C \vdash J \vee \neg \neg C$