Introduction to Symbolic Logic

David W. Agler

Predicate Logic Semantics with Variable Assignments

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Predicate Logic using Names

Recall the following valuation rules for predicate logic (let $\ _1; : : : ;\ _n$ be any series of names (not necessarily distinct), P be any n-place predicate, and $\ ;\ $ are wffs in RL):

Definition (RL-valuation using names)

1. if $P\ _1 : : :\ _n$ is a closed atomic wff in RL, then $v_M(P\ _1 : : :\ _n) = T$ iff $h\mathcal{I}(\ _1); : : : ; \mathcal{I}(\ _n)i\ 2\ \mathcal{I}(P)$, otherwise $v_M(P\ _1 : : :\ _n) = F$
2. $v_M(:(\ )) = T$ iff $v_M(\ ) = F$
3. $v_M(\ \wedge\ ) = T$ iff $v_M(\ ) = T$ and $v_M(\ ) = T$
4. $v_M(\ \_\ ) = T$ iff $v_M(\ ) = T$ or $v_M(\ ) = T$
5. $v_M(\ !\ ) = T$ iff $v_M(\ ) = F$ or $v_M(\ ) = T$
6. $v_M(8x)\ = T$ iff $v_M\ (\ =x)$ for every name $\ $ in RL.
7. $v_M(9x)\ = T$ iff $v_M\ (\ =x)$ for at least one name $\ $ in RL.

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4. $v_M(\ \ \_\ \ ) = T$ iff $v_M(\ ) = T$ or $v_M(\ ) = T$
5. $v_M(\ \ !\ \ ) = T$ iff $v_M(\ ) = F$ or $v_M(\ ) = T$
6. $v_M(8x)\ \ = T$ iff $v_M\ \ (\ =x)$ for every name in RL.
7. $v_M(9x)\ \ = T$ iff $v_M\ \ (\ =x)$ for at least one name in RL.

Two Problems: Problem 1

This definition of the valuation function has at least two problems:

Problem 1: the valuation function is only defined for **closed RL-wffs**. It is **undefined** for open RL-wffs (wffs with free variables, e.g. Px). A more inclusive valuation function might be desirable for a few reasons:

- **special wffs:** Ixx where I is the two-place identity predicate
- **compositional concerns:** shouldn't the truth value of $(\exists x)Px$ be determined by the existential quantifier and Px, rather than say a wff $Pa \vee Pb, \ldots, Pn$?
- **tighter relation between syntax and semantics:** in some systems open formulas are wffs, but they are not interpreted. A valuation function that covers open formulas would mean that all wffs are interpreted

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Two Problems: Problem 2

Problem 2: it assumes that there is an RL-name for every item in the domain.

Notice: we specify the value of existentially and universally quantified wffs in terms of non-quantified wffs. Example: $v_M(8x) = T$ iff $v_M(=x)$ for every name in RL.

the issue isn't that we don't have enough names

the issue is there is no guarantee that every object in the domain is named

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Some objects may not be named

Cats that are pets are usually named; but some stray cats may be unnamed.

The key idea

The key idea behind fixing both problems is to treat variables like **pronouns** rather than **names**.

In "It is happy" (Hx), the pronoun "it" can refer potentially to any item in the domain. Its truth or falsity will depend upon the referent of x or "it"

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Fixing the first problem: The key idea

With respect to the first problem:

We can assign a truth value to Px "he (or she or it) is a person" if there is a way of identifying the referent of the pronoun "he".

The truth value of such a wff will depend upon the referent of "he".

If "he" designates something that is not a person, then Px is false; while if it identifies something that is a person, then Px is true.

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Fixing the second problem: The key idea

With respect to the second problem:

$(9x)Px$ "someone is a person" is true iff there is at least one way of identifying the referent of "he" such that the object is a person.

In short: $v(9x)P = T$ iff $v(Px) = T$ for at least one referent of pronoun x

$(8x)Px$ "everyone is a person" is true iff on every way of identifying the referent of "he" that object is a person.

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Variable assignment

In order to fix both problems, we will introduce some additional technical apparatus.

Let's begin with the notion of a variable assignment.

Definition (variable assignment)

A variable assignment g for a model $M = \langle D, I \rangle$ is a function that assigns to each variable some object in D.

The basic idea is that a variable assignment takes each and every variable and says what object it refers to.

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Variable assignment: notation

We need a way to specify variable assignments so that it is clear which item in the domain is assigned to which variable in the language

We will use g to stand for a variable assignment

\ $g(x)$" will specify the variable assignment of x

\ $g(x)$" reads the variable assignment g that takes x as input (it will yield an item from the domain as a value).

Example

1. $g(x) = u_1$ assigns u_1 from the domain to the variable x
2. $g(y) = u_2$ assigns u_2 from the domain to the variable y
3. $g(z) =$ Liz assigns Liz from the domain to the variable z

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Relativizing the valuation function

The next step is to relativize the valuation function not merely to a model (M) but also to a variable assignment (g)

Not simply $v_M(\ )$ but $v_{M;g}(\ )$

Under this valuation function, w s are true or false with respect to a model (M) and a variable assignment (g)

Not simply $v(\ ) = T$ but $v_{M;g}(\ ) = T$

Relativizing the valuation function to variable assignments allows the valuation function not only to cover closed atomic w s but also open atomic w s (xing Problem 1)

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Valuation function for closed and open atomic wffs

This relativization allows us to formulate two different rules for atomic wffs in RL (let be any name and x be any variable):

Definition

1a if $P\ _1 \ldots\ _n$ is a closed atomic wff in RL, then $v_{M,g}(P\ _1 \ldots\ _n) = T$ iff $\langle I(\ _1), \ldots, I(\ _n)\rangle \in I(P)$. Otherwise, $v_{M,g}(P\ _1 \ldots\ _n) = F$

1b if $Px_1 \ldots x_n$ is an open atomic wff in RL, then $v_{M,g}(Px_1 \ldots x_n) = T$ iff $\langle g(x_1), \ldots, g(x_n)\rangle \in I(P)$. Otherwise, $v_{M,g}(Px_1 \ldots x_n) = F$

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Notice that we now can de ne the truth value of w s that have free variables.

Take the w Ixx where I is the two-place predicate \x is identical to x".

$v_{\mathcal{M},g}(Ixx) = T$ iff $\langle g(x), g(x)\rangle \in \mathcal{I}(I)$.

In other words, \x is identical to x" is true (relative to the model and the variable assignment) if and only if the ordered pair consisting of the variable assignment $g(x)$ and $g(x)$ is in the interpretation of I

Put even more plainly: if the objects picked out by $g(x)$ are identical to each other, then \x is identical to x" is true

Valuation function for closed and open atomic wffs

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Open w s can be assigned truth values

v(Cx) = T ("x is a cat") is true i g(x) assigns x to an item in the interpretation of C. That is, i g(x) 2 I (C).

There is still a problem!

Problem!

- valuation rules apply only to atomic w s containing either names or variables but not to w s containing both names and variables
- The valuation rule works for Pa, Lab, Px, Lxx
- BUT NOT for Lax, Lxa (names and variables)
- the valuation rule is unde ned for these w s

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Generalizing the valuation function

To solve this problem, we will need to do two things:

1. de ne the notion of aterm that includes names and variables
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RL-Term (name or variable)

First, let's define the notion of an **RL-term**:

Definition (RL-term)

An RL-term t is any name or variable in RL.

Example

1. x is a variable; therefore it is a term
2. y is a variable; therefore it is a term
3. b is a name; therefore it is a term
4. d is a name; therefore it is a term

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3. b is a name; therefore it is a term
4. d is a name; therefore it is a term

RL-Term (name or variable)

First, let's de ne the notion of an RL-term :

De nition (RL-term)

An RL-term t is any name or variable in RL.

Example

1. x is a variable; therefore it is a term
2. y is a variable; therefore it is a term
3. b is a name; therefore it is a term
4. d is a name; therefore it is a term

Denotation of an RL-term

Second, let's define and introduce some notation for the denotation of a term.

Let the expression $[\![t]\!]_{\mathcal{M},g}$ read the denotation of the term t relative to a model $\mathcal{M}$ and variable assignment g.

Next, let's define the notion of a denotation of a term

Definition (denotation of a term)

Let $\mathcal{M}$ be a model, g be a variable assignment, t be a term (name or variable). The denotation of t relative to a model and a variable assignment (that is, $[\![t]\!]_{\mathcal{M},g}$) is:

1. $I(t)$ if t is an RL-name, or
2. $g(t)$ if t is an RL-variable.

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Denotation of an RL-term: Examples

Let's look at some examples of the denotation of a term. To do this, we will need part of a model and a variable assignment. So first consider the following:

$$D : \{1; 2; 3\}$$

$$I(a) = 1 ; I(b) = 2 ; I(c) = 3$$

$$g(x) = 1 ; g(y) = 2 ; g(z) = 1$$

Now let's look at some examples of the denotation of a term

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1. $[x]_{M;g} = g(x) = 1$
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Generalized valuation function for atomic wffs

We can now combine the two valuation functions into a single valuation rule that makes us of the notion of a denotation of a term.

Definition

if t is a term, P is an n-place predicate, and $Pt_1 \ldots t_n$ is an atomic wff in RL, then

$v_{M,g}(Pt_1 \ldots t_n) = T$ iff $\langle [t_1]_{M,g}, \ldots, [t_n]_{M,g} \rangle \in I(P)$

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Examples

take Lax, an atomic wff containing the name a and variable x ("Al loves x".)

$v_{M,g}(Lax) = T$ iff $\langle [a]_{M,g}, [x]_{M,g} \rangle \in I(L)$

$v_{M,g}(Lax) = T$ iff the ordered pair consisting of the denotation of the name a and the denotation of the variable x are in the interpretation of L

"Al loves x" is true provided, relative to a model and relative to a variable assignment, the ordered pair $\langle Al, [x] \rangle$ is in the interpretation of the two-place predicate Lxy (x loves y).

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Problem 2: Quanti ed w s and names

We have a solution for Problem 1.

But Problem 2 remains. That is, we are still unpacking the truth value of quanti ed w s using names

One initial thought is to say $v_{M;g}(9x)Px = T$ i $v_{M;g}P(= x) = T$ (for some name) or $v_{M;g}(Px) = T$.

Promising approach since we have a procedure for determining the truth value of Px relative to g; namely, $v_{M;g}(Px) = T$ i $h[x]_{M;g}i \; 2 \; I(P)$. Thus, $v_{M;g}(9x)Px = T$ i $h[x]_{M;g}i \; 2 \; I(P)$

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Does not get us the right result since the variable assignment g takes each variable and assigns it a single item from the domain.

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Something is a cat

Suppose $D: \{Jon, Snickers\}$ where Jon is a person and Snickers is a cat. Notice that $g(x) =$ Jon and that $[x]_{M,g} \notin I(C)$; therefore, $v(Cx) = F$; therefore, $v(\exists x)Cx = F$.

In other words:

- we cannot specify the truth value of quantified wffs using variable assignments alone
- we need a way of specifying the truth value of a wff like $(\exists x)Px$ such that this wff is true if there is at least one variable assignment $g(x)$ such that $g(x) \in I(P)$
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Variant variable assignments

Let's introduce the notion of a variant variable assignment:

Definition (variant variable assignment)

Let be a variable and u be an item in the domain $u \in D$ of a model, a variant variable assignment g_u is a variable assignment g for a model M except that it assigns u to .

Reading variant variable assignment notation

1. g_u is read as the variable assignment g except that the variable is assigned the item u from the domain
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Example 1 of Variant Variable assignment

Example (Illustration of a variant variable assignment)

Suppose there is a variable assignment g where $g(x) = u_1; g(y) = u_2; g(z) = u_3$.
Now let's consider one variant variable assignment: $g^y_{u_1}$.

$$g : g(x) = u_1; g(y) = u_2; g(z) = u_3$$

$$g^y_{u_1} : g^y_{u_1}(x) = u_1; g^y_{u_1}(y) = u_1; g^y_{u_1}(z) = u_3$$

Notice that the only difference between g and $g^y_{u_1}$ is that $g^y_{u_1}$ assigns the variable y to u_1 instead of u_2.

Example 2 of Variant Variable assignment

A variable assignment and a variant variable assignment might be identical.

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Question

How can variant variable assignments be used to de ne a new valuation function that will deal with the problem involving quanti ed w s?

$v(\exists x)_{M,g} Px = T$ iff

there is at least one item $u \in D$ such that $v_{M,g^x_u}(Px) = T$

$v(\forall x)_{M,g} Px = T$ iff

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De nition of a valuation function using variant variable assignments

An RL-valuation | for a model M and variable assignment g | is a function that assigns to each RL-w a truth value (T or F) using the following rules (let P be any n-place predicate, $t_1, \ldots, t_n$ be a series of terms (not necessarily distinct), be any variable, ; any RL-w):

1. $v_{M,g}(Pt_1 \ldots t_n) = T$ i $\langle [t_1]_{M,g}, \ldots, [t_n]_{M,g} \rangle \in I(P)$
2. $v_{M,g}(\neg(\;)) = T$ i $v_{M,g}(\;) = F$
3. $v_{M,g}(\; \wedge \;) = T$ i $v_{M,g}(\;) = T$ and $v_{M,g}(\;) = T$
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Take the model $M = \langle D, I \rangle$, where $D = \{1, 2, 3, 4, 5\}$, $I(N) = \{1, 2, 3, 4, 5\}$, $I(O) = \{2, 4\}$, $I(a) = 1$, $I(b) = 2$, $I(c) = 3$, $g(x) = 1$; $g(y) = 2$, and all other variables are assigned 3.

$v((\exists x)Ox) = ?$

$v_{M,g}((\exists x)Ox) = T$ since there is one $u \in D$ such that $v_{M,g^x_u}(Ox) = T$

NOTE: it is not the case that $v_{M,g}(Ox) = T$ since the variable assignment g assigns 1 to x

HOWEVER: it is the case that there is a variant variable assignment g^x_u where $(\exists x)Ox$ would come out as true

Example: consider the variant variable assignment g^x_2, viz., where g assigns the variable x to $2 \in D$. On this variant variable assignment, $(\exists x)Ox$ is true. So, $v_{M,g^x_2}(Ox) = T$. And so, $v_{M,g}((\exists x)Ox) = T$

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$v(\exists x)Ox = ?$

$v_{M,g}(\exists x)Ox = T$ since there is one $u \in D$ such that $v_{M,g^x_u}(Ox) = T$

NOTE: it is not the case that $v_{M,g}(Ox) = T$ since the variable assignment g assigns 1 to x

HOWEVER: it is the case that there is a variant variable assignment g^x_u where $(\exists x)Ox$ would come out as true

Example: consider the variant variable assignment g^x_2, viz., where g assigns the variable x to $2 \in D$. On this variant variable assignment, $(\exists x)Ox$ is true. So, $v_{M,g^x_2}(Ox) = T$. And so, $v_{M,g}(\exists x)Ox = T$

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Take the model $M = \langle D, I \rangle$, where $D = \{1, 2, 3, 4, 5\}$, $I(N) = \{1, 2, 3, 4, 5\}$, $I(O) = \{2, 4\}$, $I(a) = 1$, $I(b) = 2$, $I(c) = 3$, $g(x) = 1$; $g(y) = 2$, and all other variables are assigned 3.

1. $v_{M,g}(\forall x)Nx = ?$
2. $v_{M,g}(\forall x)Nx = T$ since for every $u \in D$, it is the case that that $v_{M,g^x_u}(Nx) = T$
3. $v_{M,g^x_1}(Nx) = T$, $v_{M,g^x_2}(Nx) = T$, $\ldots$; $v_{M,g^x_5}(Nx) = T$.

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Solution to Problem 2

Recall that the problem with unpacking the truth value of $(\exists x)Px$ in terms of Pa and Pb was that there was no guarantee that every item in the domain was named

Recall also that the problem with unpacking the truth value of $(\exists x)Px$ in terms of Px relative to a variable assignment g was that there are cases where $v(\exists x)Px = T$ but $v_g(Px) = F$

What we needed was a way of referring not simply to a single variable assignment, but a number of different variable assignments

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But wait!

What about our cat example? What about Snickers?

Something is a cat

Suppose $D : \{Jon, Snickers\}$, $I(C) = \{Snickers\}$; $g(x) = Jon$. Notice $v(\exists x)_{M,g} Cx = T$ since $v_{M,g^{x}_{Snickers}} Cx = T$ since there is a cat but $[x]_{M,g^{x}_{Snickers}} \in I(C)$

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We saw that using names to unpack the valuation function had two potential problems:

Problem 1: it left open w s unde ned. We solved this by relativizing the valuation function to variable assignments

- Allows us to specify the truth value of w s like Lxx
- tighter relation between syntax and semantics: if a formula is a w , then we have a way of assigning it a truth value

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Problem 2: it assumes that there is an RL-name for every item in the domain. We solved this by relativizing the valuation function to variable assignments and making the truth of quantified wffs depend upon variant variable assignments

- Not a problem if every item in the domain isn't named
- variables can ensure that each item is referenced in some way
- Treats the truth value of wffs in a pretty compositional way: $\exists(x)Px$ is determined by the existential quantifier and Px, rather than say a wff $Pa \vee Pb, \ldots, Pn$?

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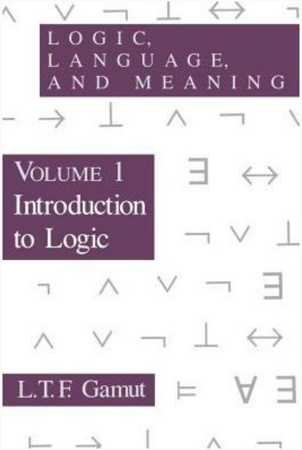

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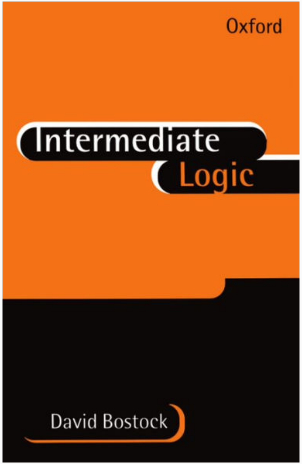

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